White Knuckle Ride

An Introduction to Dynamics

This problem introduces students to dynamics.

Intended Learning Outcomes

By the end of this activity students should be able to:

- Distinguish between static, kinetic and rolling friction
- Set up and analyse equations of motion in linear and circular motion, using approximations where appropriate
- Use kinetic and potential energies
- Calculate work done through friction

KEYWORDS:

Dynamics, static friction, kinetic friction, rolling friction, equations of motion, linear motion, circular motion, kinetic energy, potential energy, work done.

Contents

Reading List

The following textbooks are suggestions, other equivalent textbooks are available:

- Breithaupt, J. *Physics*. Palgrave Foundations.
- Tipler, P.A. *Physics for Scientists and Engineers*. Freeman.

Problem Statement

Memo:

Chypsis Toys Ltd

Our marketing department see a niche for a gravity powered roller-coaster toy of some sort: nothing too complicated – just a straight track for example. In order to convince potential investors we need some good calculations as to what can be done. We've had some prototypes of the cart put together, but I don't want to bend any tracks until after I've seen your report and we've some idea of what would work. What are the sources of friction? What heights do you think we can use for successive humps? How many humps should we have? I don't think the friction on these rails is particularly low; would it be better if we used a Teflon coating? Would your results scale if we were to decide to produce a suite of toys? At this stage I'm not interested in cosmetic aspects – we'll get the design guys to deal with that.

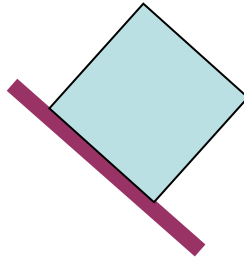I'll need a formal report from you with your results.
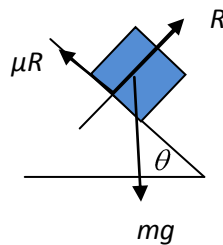
Roberta

Suggested Deliverable

Individual Report

Questions for Class Discussion

1. Does the box tumble or slide? List the assumptions you make.

2. If the frictional force is assumed to be independent of velocity show that the effective acceleration of a block on a plane inclined at an angle θ to the horizontal with coefficient of friction μ is $g(\sin θ - μ \cos θ)$.

3. (a) Show that the equation of motion for the speed v of a body of mass m rolling down a plane inclined at angle θ to the horizontal under a frictional force of the form $F_{\text{fric}} = -mrv$ with r a constant is

$$\frac{dv}{dt} + rv = g \sin θ \tag{1}$$

(b) Confirm (by substitution in (1) or otherwise) that a solution of the equation is

$$v = \frac{g \sin θ}{r}\left(1 - e^{-rt}\right)$$

(c) By expanding the exponential as a power series show that

$$v \approx g \sin θ(t - rt^2/2)$$

to first order in r, and hence, by integrating, that the distance traveled down the slope in time t is approximately

$$s = \frac{g't^2}{2} - \frac{rg't^3}{6}$$

where $g' = g \sin θ$.

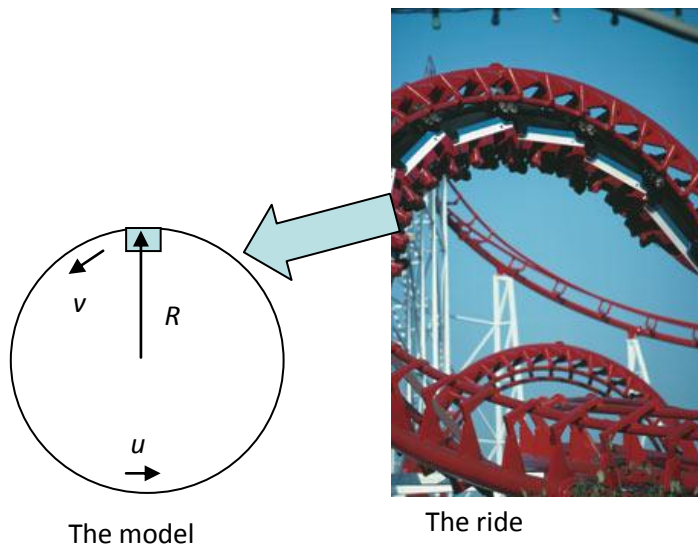(d) Verify (by substitution for v or otherwise) that to this order

$$s \approx \frac{v^2}{2g'} + \frac{rv^3}{3g'^2}$$

(e) How could you use the results in (c) and (d) to investigate the applicability of this form for the friction in your experiments?

4.   Show that the work done by rolling friction of the form F_{fric} = -mrv from rest to a speed v is

$$\frac{mrv^3}{3g'}$$

5.   Assuming that the carts are not attached to the rails, does the g-force on the riders limit the diameter of a circular track in a fairground ride?

The model   The ride

6.   What effect would a teflon coating have on the design of the toy track for **Chypsis**?

7.   Can you scale the model (up or down i.e. multiply all lengths by a constant factor, all masses by a factor and so on)? To answer this, show that if the frictional force is rv, independent of mass, with r a constant then (to first order in r) the ratio of successive heights h_2/h_1 is

$$\frac{h_1 - \frac{r}{3gg'm}(2gh_1)^{3/2}}{h_1}.$$

Show that this is independent of the scale provided that the mass of the cart is proportional to $h_1^{1/2}$. [Hint: consider the work done and use question 4]

Can the ride be scaled at all if the friction is mrv? Is air resistance important in full scale rides?